Modeling, solving & implementing PDEs from wave-structure interactions.

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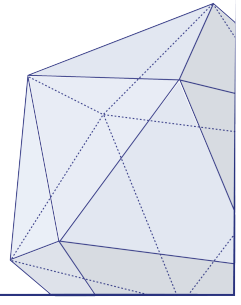


Title overview

Modeling, solving & implementing

Tools.

- Considering physics and studying Shallow-Water models;
- Designing high-order numerical schemes to solve equations;
- Development of a numerical simulation to test our schemes.

Goal. Using everything to describe

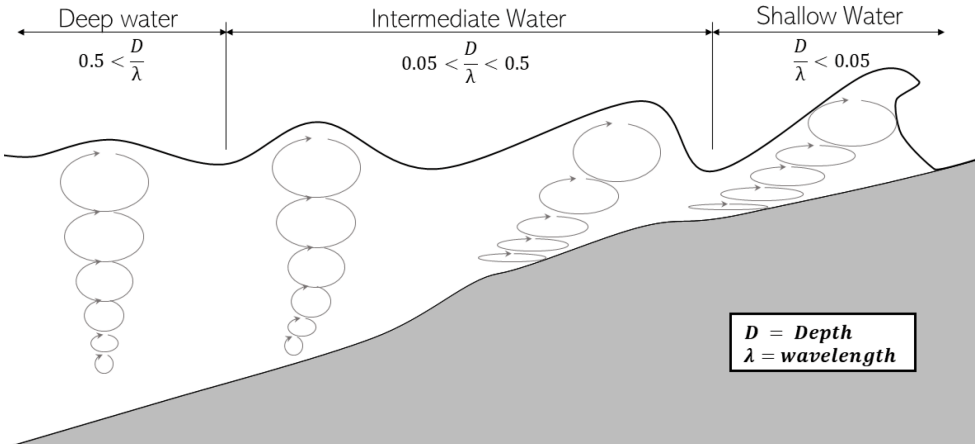
wave-structure interactions

Shallow-Water model

Non-linear hyperbolic system with source term:

$$\partial_t \mathbf{V} + \nabla \cdot \mathbf{F}(\mathbf{V}, b) = \mathbf{B}(\mathbf{V}, \nabla b),$$

with $\mathbf{V} = (\eta, q_x, q_y)^T$ (η: water-height, $\boldsymbol{q}$: horizontal discharge).



- Describe hydrodynamics on coastlines, lakes, nearshore;
- Derived from Navier-Stokes equations, allowing efficient big-scale simulations.

⚠ Hyperbolicity means exact solution can be **discontinuous** !

Combine FV & DG approaches

For hyperbolic problems, we mainly use two types of numerical schemes: Finite-Volume (FV) & Discontinuous Galerkin (DG).

Finite-Volume
(+) Robust, easy to implement.
(−) Not very precise (order 1).

Discontinuous Galerkin
(+) Arbitrary precise (order k).
(−) Less robust, harder to implement.

Then why not combine both ?

Introducing a scalar $\Theta \in [0, 1]$, we can construct a robust and arbitrary precise hybrid scheme:

$$Hyb = FV + \Theta\ (DG - FV),$$

Θ being computed according to any convex properties we want.

Applications to wave-structure interactions

Wave-structure interaction is a *recent* and *challenging* research domain, it means a **lot of work** has to be done ☹

1. Coupling Shallow-Water equations with a floating object;
2. Adapt our hybrid *DG-FV* scheme in order to solve our system;
3. Implementing everything to obtain a numerical simulation.

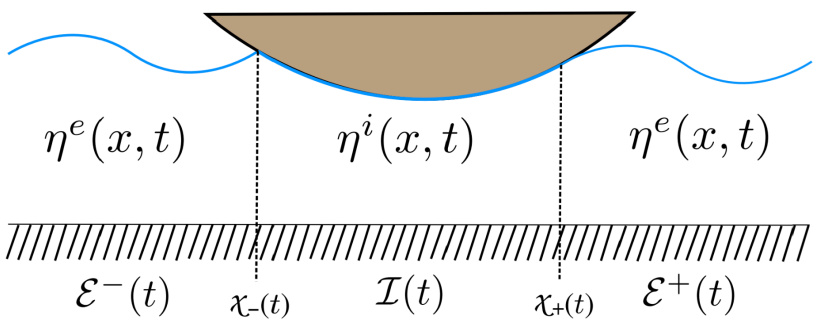


Figure: Wave-structure complex.

Our work will be obviously useful for a *mathematical purpose*, but will also serve larger interests, especially with the development of *renewable energy*.

Figure: *The Great Wave of Kanagawa*, Hokusai, 1830.

Thank you for your attention !